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and our readers may be interested in verifying the true result. If $\sqrt{PR/ES} = \alpha$, then α should be replaced by the next smaller integer n or the next greater integer $n + 1$, not according as α is less or greater than the arithmetic mean $n + \frac{1}{2}$, but according as α is less or greater than the geometric mean $\sqrt{n(n + 1)}$. For large values of n, the distinction between the two criteria is slight. Of course in any actual case it would be simple enough to substitute each of the two values in the expression for C in order to determine which was the better.

I. The Mathematics of Biometry.

By Lowell J. Reed, Johns Hopkins University.

(Read before the Mathematical Association of America, January 1, 1920.)

I think we are all agreed that the program for this meeting of the Mathematical Association of America is as important as any that the Association has ever discussed. Any science that fails to ally itself with the other sciences must of necessity have a narrow development, and this is perhaps more true of mathematics, due to its wide application, than of any of the other sciences. It is to be hoped that the Program Committee will carry out its own suggestion that at some later meeting we consider the converse of the present question, that is, the contribution of other sciences to the development of mathematics.

My own part in the present program is the discussion of the mathematics of biometry, and instead of presenting illustrations of the application of mathematics to this branch of biology I am going to outline what I consider to be the proper mathematical training for work in biometry. I wish first to call attention to the development of the present method of teaching mathematics. We have in the field of mathematics a number of branches that have been developed mainly as tools for the solution of practical problems. Thus, trigonometry has been developed mainly for the solution of problems in surveying; probability has been developed for games of chance. Now whenever this has been the case the teaching of the subject has been concentrated on this one particular application, to the exclusion of all others. In recent years the increasing use of mathematical methods in such sciences as chemistry, biology, etc., has led to an effort to broaden the teaching of mathematics by introducing new applications. The result has been a new group of textbooks under such titles as "Calculus for Chemical Students," "Mathematics for Agricultural Students," etc. All of these texts seem to me to miss the point in that they imply that calculus for chemistry students is distinct from calculus for other groups. This I do not believe to be the case. If we consider the mathematical needs of a student in any one of the sciences we find them about as follows: First, he needs a foundation in algebra, trigonometry, analytic geometry, and calculus; and secondly, he needs to be trained to take a problem in his particular field and translate it into mathematical language. The latter need is the greater of the two, and it is the one that it is the more difficult to satisfy.

I now wish to present what seems to me to be the proper mathematical training for work in biometry; although the following courses are outlined with the needs of a student of biology in mind, I feel sure that they are the proper ones for students in any branch of science. The first year courses in mathematics should cover the three subjects, algebra, trigonometry, and analytic geometry. The course in algebra should be fully as complete as the one at present given and may occupy the first half of the year. Emphasis should be laid on problem solution with the idea of satisfying the second of the needs listed above. The course in trigonometry should give to the students a knowledge of the trigonometric functions, showing their relations to each other and to the right-angled triangle; it should develop the relations that exist between the function of the angle and the sides of the general triangle, but it should not contain the usual extensive drill in the application of these formulas to the solution of triangles. The use of the Law of Sines and Law of Cosines in mechanics is fully as important as their use in solving triangles and the application is not obscured by a long computation, but one very seldom sees such applications in a textbook on trigonometry. The course in analytic geometry should give the methods of the subject with a fairly complete discussion of the straight line and circle, and a brief discussion of conics. In this course again there is an excellent opportunity to train students in the methods of expressing ideas in mathematical notation. To the first year's work might well be added a short section on computation so that the students could become acquainted with the use of logarithms, and if possible learn to use some of the various calculating machines.

The second year's work should be divided into two parts. The first half year should be devoted to a course giving the elements of calculus, emphasizing principles and methods rather than the tricks of differentiation and integration. In the second half of the year the students should be divided into groups according to their interests, one group for those majoring in engineering, another for those majoring in the natural sciences, etc., the divisions formed being adjusted to suit the needs of the institution in question. Then the students in each group should be given work applying the preceding mathematics course to their respective fields. The students majoring in different natural sciences should not in my opinion be further separated into distinct groups as they are all concerned with the same fundamental problem, that is, the analysis of observed data, and the course given to this group of students should train them in modern statistical methods of analysis. First, the subject of probability should be presented from its theoretical side. This should include the fundamental definitions and a discussion of probability theory leading to the normal curve. The meaning of the constants of the normal curve and the notion of probable error should be introduced at this point. The subject of probability should be followed by a discussion of the principle of moments, illustrations being drawn from mechanics. The treatment of this subject should lead naturally into a discussion of frequency distributions and correlation. The relationship of the first, second, and third moments to the mean, standard deviation, and the skewness of frequency distribution, should be

clearly brought out, mechanical illustrations being freely used. The correlation coefficient should be developed as a product moment with the mean of the variables taken as an origin. Following this the students should be taught the theory of curve fitting by the method of moments and this theory should be illustrated by examples in the derivation of empirical formulas. The method of fitting curves by least squares need not be presented as it is not so widely applicable as the method of moments. The student should also have impressed upon him the importance of the idea of probable error and should be trained to work out the probable error of all such constants as mean, standard deviation, coefficient of correlation, etc.

The course just outlined would furnish a student in biometry with the proper mathematical foundation for his work; his mathematical needs are similar to those of the student of any natural science. One of the most common quotations among biometricians is that of Sir Francis Galton: "Until the phenomena of any branch of knowledge have been submitted to measurement and number it cannot assume the status and dignity of a science," and as we see more and more of the leading scientists taking this point of view it impresses upon us the growing importance of mathematics. The methods of teaching mathematics should develop accordingly; and if this present meeting succeeds in stimulating the teaching of mathematics to live up to its increased importance, it will indeed have been a success.

II. Complex Numbers in Advanced Algebra.

By H. E. Webb, Central High School, Newark, N. J.

In many electrical laboratories it is desired that the workers have a certain familiarity with vectors, and in particular with the exponential notation $e^{i\theta}$ Examination of texts in algebra most commonly used in schools fails to bring to light an elementary explanation of this notation. Certain texts in trigonometry give such an explanation, but usually in a rather intricate form, and without a clear statement as to what it is all about. The following brief outline is suggested as an addition to a course in advanced algebra for the fourth year in high school or the freshman year in college; in this it is desired to escape from exhaustive analysis of various points which seemingly of necessity must be brought into any *rationale* of this notation.

1. The rules of combination of complex numbers to be presented as an arithmetic of number pairs, *without reference to Argand's diagram.*

2. The sine and cosine of *real positive and negative numbers* to be defined by reference to a unit circle, without mention of other trigonometric functions, leading only to the facts that in a circle of radius r the corresponding lines are $r \sin \theta$ and $r \cos \theta$, and that $\sin^2 \theta + \cos^2 \theta = 1$ for all values of θ.

3. The development of formulas for $\sin (x \pm y)$ and $\cos (x \pm y)$.

4. The polar notation of a complex number, shown as follows: If $a + ib$ is